Exploring Self-Organised Criticality in an Agent-Based Ecological System: Extensions on the Bak-Sneppen Model

Matthew A. Ford

Department of Informatics, University of Sussex, BN1 9QH, UK

mf472@sussex.ac.uk

Abstract – This report builds upon the basis of the Bak-Sneppen model to explore whether the same traits of self-organised criticality (SOC) can emerge in complex, agent-based ecological system with predator-prey dynamics and a structured food-chain network. The model incorporates trophic-level species, adaptive reproduction and mutation, and resource-based survival with biomass competition. The model was found to exhibit SOC in some cases, but others failed to meet the power-law distribution. This leads to the conclusion that with more complexity comes further risk of pushing the system away from a critical state.

1. Introduction

Self-organised criticality (SOC) has been a long-lasting challenge to the conventional understanding of systems science. This concept is a unique phenomenon where complexity and order emerge spontaneously without external tuning. *(Bak, P. et al., 1987)*. In this context, self-organisation refers to the system naturally evolving to a critical state, without a detailed specification of the initial conditions - this point acting as an attractor in the system's dynamics. *(Bak, P. et al., 1988)*.

One of the most well-known models in the field of SOC in evolutionary dynamics is the Bak-Sneppen model. *(Grinfeld, M, et al., 2010)*. This model simulates an ecosystem where the species are arranged on a lattice, and each assigned a random fitness value. At each time step, the species with the lowest fitness, along with its direct neighbours are assigned new fitness values. *(Bak, P. et al., 1993)*. This iterative process eventually leads to cascades of extinctions, as neighbouring species of an extinct organism are given new random fitness values and may themselves become the least fit. These system dynamics and extinction events – also known as "avalanches" – are characteristics of SOC in the Bak-Sneppen model.

Another feature of SOC present in the Bak-Sneppen model is the avalanches of activity that are power-law distributed. *(Felici, M. et al., 2001)*. In this context, the power-law distribution is where the probability of an avalanche is decreased as a power of its magnitude

Meester and Sarkar (2012) provided proof through their modified version of the Bak-Sneppen model that a power-law behaviour in avalanche size and duration is present. This furthermore strengthens the alignment of the model with SOC characteristics.

These concepts of SOC, particularly from the Bak-Sneppen model have inspired the development of more complex systems involving evolutionary dynamics. While the Bak-Sneppen model provides a simple abstraction of ecosystem evolution, modern tools such as agent-based modelling (ABM) can be used to create more detailed representations of these systems and their dynamics. MESA is a leading framework for this.

Mesa, a Python-based ABM library *(Masad, D., et al. 2015)* is a leading framework that allows researchers to create ecosystems populated with autonomous agents. These agents can be arranged onto networks and directed graphs which in turn enables more realistic ecological relationships to be simulated, such as predator-prey and food chain hierarchies. To represent and analyse these complex ecological networks, NetworkX, a Python library for the creation, manipulation and study of complex networks *(Hagberg et al., 2008),* can be used alongside Mesa. This provides the tools needed to construct directed graphs for food-web representation. This facilitates the simulation of real-world interactions in ecosystems, extending beyond the lattice structure of the Bak-Sneppen Model.

More specifically, food webs can be represented as directed graphs where the nodes correspond to species, and the edges represent the energy flow across trophic levels. Unlike the Bak-Sneppen model's lattice structure, these graphs can capture heterogeneity and complexity of real-world ecosystems *(Dunne et al., 2002)*.

ABM enables the incorporation of adaptive behaviour, in this case through mutation and reproduction of species. This offers a powerful platform for studying how SOC-like dynamics can emerge in certain systems *(Grimm et al., 2006)*. By exploring models in Mesa, it is possible to investigate the effects of species traits, network topology and inter-species dependencies affect the emergence of SOC.

Adaptive systems change and reorganize their component parts to adapt themselves to problems posed by their surroundings. *(Holland, J., 1992)*. In the case of the proposed model this involves self-organisation, mutation, reproduction, weighted networks and feedback loops between the agents and their environment.

In this report, an extension to the Bak-Sneppen model is proposed. This incorporates trophic-level structure, adaptive behaviours – for repopulation, extinction and resource availability – and network-based species interaction. By integrating ABM with network analysis tools, the model's main aim is to simulate ecosystems with greater biological realism, but not to the extent of full biological models. This allows for the exploration into how SOC emerges from complex and adaptive species interactions.

The main characteristics of SOC are the scale-invariant cascades and robust power-law scaling of avalanches. *(Planz, D., et al., 2021)* Also, the system should be robust to damage due to small perturbations in initial conditions. *(Stapleton, M. et al., 2004)*. Meaning that the parameter values should not divert of push the system into a state of SOC.

This has the aim of closing the gap between simple, abstract ecological models and real-world complexity. Offering insights into extinction dynamics and adaptation.

2. Methods

The model created extends on the Bak-Sneppen model by incorporating more ecologically realistic dynamics by using ABM and network-based species interactions. This model is implemented in Python using the Mesa and NeworkX libraries. It simulates an ecosystem where the species are represented as agents, which are then nodes connected through a directed graph resembling a food-web.

Unlike the original Bak-Sneppen model, the proposed model employs a weighted directed graph structure to capture trophic interactions and energy flow through the system. This section outlines the model architecture, agent design, network structure and generation, fitness functions, and simulation procedures used to observe any emergent behaviours.

2.1 | Agent Design

To simulate ecological systems, a single species class was created with four possible types: carnivore, herbivore, omnivore or producer. The assigned type determined the traits and randomly initialised value ranges for each individual species as well as their energy sources.

Each agent has their own set of traits. For the producers this is their production rate, and for the other species, these are: strength, speed, stealth, and metabolism, all of which influence its survival, energy acquisition and interaction outcomes. The traits are sampled from set ranges that help to introduce noise to the system and to influence the directionality and weights of the trophic links in the interaction graph.

While producers convert biomass to energy, limited by their production rate, all non-producer species expend energy through their metabolism.

Agent interactions are embedded using a directed network. Here, edges represent the predation links. These links have weights that are initialised using the traits reflect the quality of their prey (as will be discussed in *Section 2.2*), this introduces adaptive interaction dynamics as the network changes forms due to cascading extinctions and the emergence of SOC traits.

2.2 | Network Structure

The Bak-Sneppen model uses a standard lattice structure to simulate the interactions. Here, extinct species influence their direct neighbours – which is maintained in this model – but links are undirected and uniform which is not typical for ecological systems. Therefore, it was decided that a directed graph was to be used as this could simulate directed predation. This was implemented using Mesa's 'network grid' module alongside NetworkX which were discussed in *section 1.*

Figure 1 shows a systems diagram which demonstrates the ecological dependencies between species and their environment within a dynamic food web.

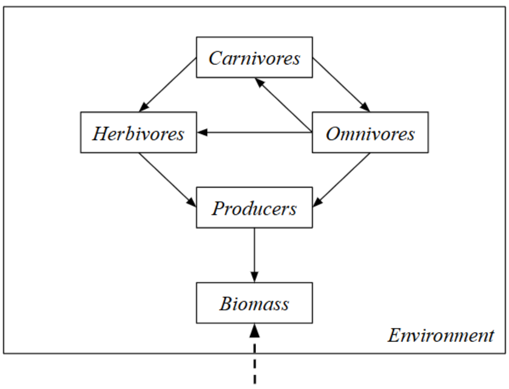

Figure 1: A systems diagram showing the trophic dynamics of the system, contained within its environment.

The producers are the sole agents capable of converting environmental biomass directly into energy. This is governed by their production rate parameter, and the draw from a globally regenerating biomass store. This makes them the foundational energy source for the entire ecosystem.

Herbivores acquire energy exclusively from producers. Their survival is therefore closely linked to the energy availability from producers and are subjected to direct competition both with each other and omnivores for access to this limited resource.

Omnivores are both consumers from producers and other species. This allows then to extract energy from any species type and as a result makes them the most resilient and adaptable class in the ecosystem.

Carnivores are solely predators; they only gain energy from herbivores and omnivores. Consequently, their survival is indirectly dependent on the size of the producer populations which affect the availability of prey. This simulates the interdependencies described by the Lotka-Volterra predator-prey model.

During initialisation, predator connections are established randomly while maintain a rule that disallowed predation within the same species type. This increased the interdependence among species and enhanced the potential for extinction avalanches which is associated with SOC.

Each predator-prey connection had a weight assigned based on the relative differences in the agent traits described in section 2.1. Specifically, the initial weight, w, of a link is computed using the following formula:

$$w = \frac{1}{1 + e^{-(\Delta_{speed} + \Delta_{stealth} + \Delta_{strength})}}$$

This formula was applied to all species other than the producers, as they are static agents that only have a production rate trait. It aims to mimic the ecological intuition that faster, stealthier, and stronger prey are harder to catch and therefore provide less energy per step in the model.

After each extinction avalanche, it was necessary to adjust the weights globally to reflect the current predation pressure on each prey. In the situation where a prey node has many incoming connections, the weight of each individual link is reduced to simulate competition amongst predators. The formula for this is as follows:

$$w_{new} = w_{old} \cdot (1 - \frac{In_{connections}}{50})$$

$In_{connections}$ in this case represents the number of predators currently predation on that prey. This allows the system to dynamically adjust the energy flow through a feedback loop which is a property that encourages SOC behaviour and enforces the adaptive nature of the system.

2.3 | Fitness and Extinction Dynamics

Fitness in the Bak-Sneppen model is handled through random assignment. When a species becomes the least fit, it and its neighbours are assigned new random fitness values, simulating extinction. While this exhibits SOC, it is not ecologically accurate.

In this proposed model, fitness is derived from ecological interactions given by the network structure from section 2.2. Each agent has a fitness hat is based on the benefit, b, given from its prey and producers, and is offset by a predation pressure, p, from any predators. This is defined as follows:

Let:

- $\boldsymbol{S_c}$: The set of non-producer prey of agent i.
- $\boldsymbol{S_p}$: The set of producer prey of agent i.
- $\boldsymbol{P}$: The set of predators of agent i.
- $\boldsymbol{w_{i,s}}$: Edge weight from i to prey s.
- $\boldsymbol{w_{p,i}}$: Edge weight from predator p to i.

Then:

$$b = \sum_{s \in S_c} \min(1.0, \frac{1}{w_{i,s}}) + \sum_{s \in S_p} \min(1.0, \frac{1}{w_{i,s}})$$

$$p = \sum_{p \in P} w_{p,i}^{1.5}$$

$$fitness_i = b - p$$

Having this formula define fitness allows for both the quality (weight), and number of trophic interactions to be considered.

To preserve the foundations of the Bak-Sneppen model, only the single species with the lowest fitness is removed at each time step. Following this, the interaction graph is reassessed, and all fitness values are recalculated. If any of the direct neighbours of the extinct species subsequently becomes the least fit, the extinction avalanche continues with their removal. This process repeats iteratively, propagating extinctions through the network until there are no further species that meet the requirements for extinction. Once the cascade halts, all extinct agents are then replacing to maintain the population and avoid complete extinction.

An additional feature of the proposed model is the ability for agents to reproduce. This is governed by a reproduction rate along with a species energy availability. If a single species has an excess of energy, it may reproduce, using half of its available energy to create the offspring. The resulting child inherits the parent's species type and either replicates its traits or is given slightly mutated variants to introduce variation.

Once created, the offspring is added to the interaction graph and assigned new predator and prey links, following the defined rules. This mechanism introduces a system of continual structural changes and noise, promoting the conditions for SOC. As resilience to system deviations is a trait of SOC, the model's ability to maintain critical dynamics through fluctuating populations helps to give insight into the adaptive self-organisation aspect.

2.4 | Biomass and Energy Flow

Biomass in the model is the only regenerating resource and the foundational input for energy flow throughout the ecosystem. As seen in Figure 1, biomass is the only component with an external input. This is a tuneable parameter that determines how much biomass is introduces into the pool at each time step.

Figure 2 illustrates the energy transfer into and through the system.

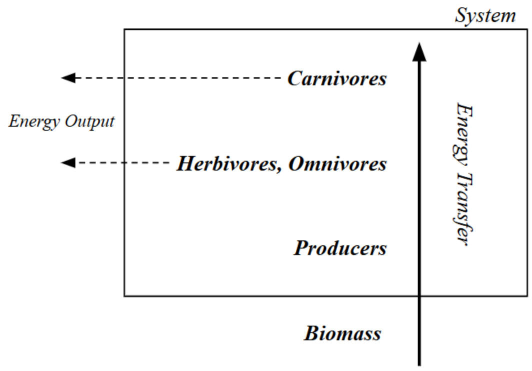

Figure 2: Energy Input, Transfer and Output through the Modelled System.

As discussed in *section 2.2,* only the producer species can convert biomass into usable energy, acting as an entry point of energy into the food web. The energy intake, however, is regulated by the production rate of each producer, and the total energy influx per step is calculated as:

$$Energy_{step} = \sum_{p \in P} production\_rate_p$$

From here, energy is distributed throughout the ecosystem using the directed food web graph. Predators extract energy from prey, and herbivores or omnivores feen on producers. This structure establishes the trophic levels and ensures that the energy flows through inter-species interactions rather than being arbitrarily distributed across the population by allowing all species to extract from the source.

As mentioned in section 2.3, energy also is a critical factor in the population dynamics and reproduction. This is because reproduction is only allowed when a species has stored a surplus of energy, simulating ecological constraints such as resource scarcity.

At every step, and as seen in *Figure 2*, energy is dissipated by all non-producer species. This is due to their metabolism, a value that is individually specific and randomly initialised between 0.9 and 1.5. This constant energy loss forces species to maintain strong links to other agents with energy stores to avoid becoming the least fit, and to keep reproducing. This helps keep the pressure in the system and allow for drops in energy to potentially cause larger extinction avalanches.

2.5 | Model & Scheduling

This model uses Mesa's 'RandomActivation' scheduler. This controls the order in which the agents are activated at each simulation step. Using this scheme, all agents are allowed to act exactly once per time step but the order in which they act is at random. This is used to avoid deterministic behaviours that arise from patterned behaviour and eliminate biases in the simulation.

Random activation is particularly favourable for investigating SOC as these systems have cascades that are triggered by local changes that propagate through the system. When randomising the order of which agents' step, the noise of the system is maintained which helps to avoid cyclic behaviours from emerging and supressing the characteristics of SOC.

The model makes use of Mesa's 'NetworkGrid' which is a graph-based environment in which agents can be positioned onto the nodes of a directed network. This allows for the graph in section 2.2 to be the base representation. The functionality of the network grid module means that it is simple to add/remove edges and nodes, track any neighbours and allow for the dynamics of the graph to be adaptable through rewiring.

This choice was seen as the most logical for capturing and maintaining the structure of ecological interactions instead of relying on proximity on a 2D grid. This further aligns with the Bak-Sneppen model as it allows for local interactions without the need for luck that two agents cross paths. Again, this maintains the core principles of an SOC model, whilst having a more detailed and flexible representation.

2.6 | Simulation Parameters

The following parameters were chosen to be specified and determine the quantitative dynamics of the system. However, they are not intended to fine tune the emergence of SOC. These parameters include:

- **Number of Agents:** The starting number of agents for the model. These are randomly initialised to the species types. For this experiment, the values ranged from 20 to 170 in increments of 30.
- **Reproduction Rate:** This, along with the energy surplus, determine if an agent can reproduce. The values range from 0.1 to 0.4, with increments of 0.075.
- **Mutation Rate:** This determines the probability that the offspring's traits are altered slightly from that of its parent. This helps to add more diversity in the system. The values ranged from 0 – 0.15.
- **Biomass Regeneration Rate:** This is the value added to the biomass pool at each time step. Ranging from 50 – 175 in increments of 25, having a higher regeneration rate would ensure that the energy input into the system per time step is always high.
- **Biomass Start:** This value ranges from 200 – 800 and determines the amount of biomass in the pool as the model is created. With a low start and regeneration rate, this can prohibit the population size drastically.
- **Epsilon:** This determined the tolerance of the system for making extinctions happen. The lower this value is, the closer the fitness value must be to the current global minimum for the species to go extinct. This could potentially prolong or shorten avalanches. The values here ranged from 0.001 – 0.1.

In order to maintain the principles of the Bak-Sneppen model and SOC, the parameters are used to create unconstrained environments in which SOC can emerge naturally. As SOC is defined to self-organise towards a critical state, this means that the system should be robust and not too sensitive to the parameter value changes.

Therefore, rather than forcefully pushing the system into a critical state, the goal of these parameters is to show whether the system exhibits any traits of SOC, such as resilience to perturbations, scale-invariant cascades, and power-law distributed avalanches. *(Stapleton, M. et al.. ; Plenz, D., et al.)*. Evaluating the model over these ranges helps to evaluate its ability to maintain these properties and decide whether there are signs of true SOC.

2.7 | Data Collection

The data is collected through a parameter sweep of every combination. To speed up this process, multiprocessing was used. This allowed multiple models to be run simultaneously and allowed for a larger parameter sweep to be run with a much greater efficiency.

As mentioned, the parameter values are not used to force SOC but instead are used to reinforce any possible properties of SOC being exhibited. Therefore, the following metrics were tracked for data to show any of these properties along with the system dynamics:

- **Energy at Each Time Step**
- **Average Energy**
- **Biomass at Each Time Step**
- **Average Biomass**
- **Population Counts:** The number of each species type at any given time step.
- **Average Species Counts**
- **Avalanche Sizes** (any time step)
- **Avalanche Durations:** How long the avalanches continued for after the initial extinction

Avalanche sizes and durations are the most critical metrics to detect SOC. The duration of both can be plotted on a log-log scale. If a straight line is seen, then that indicates a power-law distribution. These can also be fit to a power law model to obtain qualitative data on the fit quality and slope.

The population metrics provide a way for long-term behaviours to be identified, and the averages can be used to show if the system values are exploding and therefore can offer an insight into the ecological plausibility of any criticality observed.

2.8 | Adaptivity in the Model

This system has multiple aspects that can help not categorise it as an adaptive system. Here, adaptivity arises from several interconnected components.

At each time step, a fitness function evaluates each of the species, mostly through their trophic interactions. Low-fitness species are removed through an extinction process which can cascade through the network. This is reflective of a natural selection scenario, where the system adaptively prunes the ecosystem.

The food-web network topology is not static in this model. As species go extinct, new ones are introduced and connections representing predator-prey links are redefined. This makes sure that the network can restructure to account for ecological shifts. Having the network evolve based on local dynamics is a supporting factor in the system's adaptability.

Finally, the dependency on energy for species to reproduce introduces a form of feedback loop: agents that tend to have lower metabolism and better predation links survive longer to reproduce. As a result, the dynamics of the system adapt to reflect the availability of biomass and energy as well as the inter-species interaction.

Together, these mechanisms work to create a system where species, populations, and network topology adapt and co-evolve in response to fitness, resource fluctuation and interaction dynamics.

3. Results & Analysis

The output from the parameter sweep included data on system-level averages, such as energy, biomass, population measures alongside the necessary data to analyse SOC, such as avalanche sizes and durations. These help to capture both the ecological stability and any emergent dynamics.

3.1 | Evidence of Criticality

To assess the emergence of SOC, the distribution of avalanche sizes was tracked through multiple runs. As SOC follows a scale-invariant behaviour that can be identified through power-law distributions, visualising these distributions was necessary.

The formula for a power-law distribution is as follows:

$$f(x) = a \cdot x^{-k}$$

As seen in *Figure 3 (left),* the top five model runs exhibit a perfect linear relationship (R^2 $Value$) when plotted on a log-log scale – which is achieved by taking the log of both sides of the power-law distribution formula. These show that the avalanches were following a power-law distribution and therefore indicates strongly that the system was in a state of SOC.

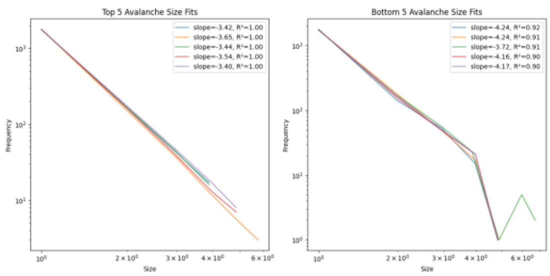

Figure 3: The Top 5 (left), and Bottom 5 (right) Avalanche Size Distributions Plotted on a Log-Log Scale

The critical exponent (k) in these figures are representative of the slope of each line and can vary depending on the model. For example, in some forest-fire models, this exponent is found to be ≈ 2.15 *(Clar, S., et al., 1996)* and for some sandpile models, this is found to be ≈ 1.51 *(Kutjnak-Urbanc, B., et al. 1996).*

Therefore, for the best-fitted results, having an exponent that is ≈ 3.4 is plausible and suggests that the model self-organises towards a critical state.

The emergence of a power-law in the best performing runs suggest s that the system achieves a balance between order and chaos, enabling varied sizes of disturbances whilst avoiding complete collapse. This indicates that there is ecological resilience and starts to show that this pattern naturally emerges from the system structure and feedback without the need for manual fine-tuning. But this can also start to be disproven with some other results.

As seen in *Figure 3 (right),* the model runs with the lowest R^2 value deviate from a pure power-law distribution. Although the absolute lowest value is 0.9, this is still a large enough deviation to show that the system had not entered a completely critical state. Here, the slope value (k) ranges from 4.16 to 4.24. This combination of steeper and inconsistent slopes indicate that the scale-invariance had been broken which could have been due to the system dynamics preventing avalanches.

However, this could also be a result of the set time strep limit of 2000. This would generally be long enough for models to self-organise towards the critical state, but in some cases, this may have taken longer and therefore the data was collected too early. This is something that would be impossible to tell as defining parameters for criticality before the run terminates could very easily cause near-infinite run-times.

3.2 | Ecological Dynamics

Figure 4 displays the species population dynamics for the top 5 runs (outlined in *Figure 3*). Here, it is shown that all species have persistent population fluctuations which indicate that the system is dynamically stable. This idea is reinforced as there are no populations that crash or explode, omnivores mostly retain the highest population, followed closely by carnivores and herbivores. Producers maintain a lower population but still exhibit recovery cycles.

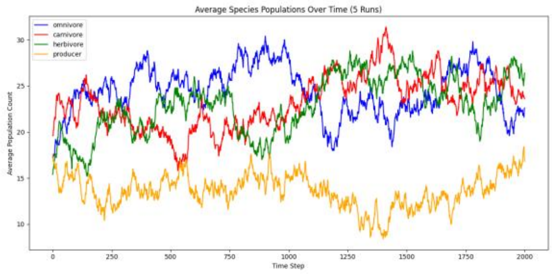

Figure 4: Average Species' Populations Over Time for the Top 5 Runs (Determined by the R2 Value)

The trophic interactions can be interpreted from this plot as it appears that as the herbivore population dips, the producers often recover – something often seen with predator-prey dynamics. This pattern can also loosely be identified between the omnivores and herbivores.

As with the nature of the Bak-Sneppen model, no species ever goes extinct as they are replaced upon extinction. However, even with reproduction as an additional mechanic, the populations avoid explosion or single-species domination. This reinforces the idea of the system staying on the edge of chaos as these dynamics are not ordered yet not chaotic.

These fluctuations across all species show that the model is nearing a critical point, this contributes to the emergence of avalanches and SOC properties.

Figure 5 shows the energy and biomass averages over the same 5 runs. As expected, these are inversely linked, meaning where the biomass dips, the energy peaks. This is expected from the consistent conversion of biomass from the producers.

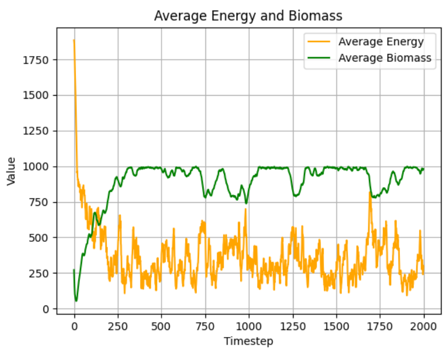

Figure 5: The Average Biomass and Energy for the Top 5 Runs (Determined by the R2 Value)

An important note is that despite the consistently high biomass available, the population levels shown in *Figure 4* remain bounded and continue to fluctuate within a specific range. This reinforces the idea that there is a bottleneck effect for the energy distribution into the system. This means that energy acts as a slow, driving force that gradually accumulates until there are larger extinction cascades. This is like the sandpile model where small build-ups lead to a larger release (avalanche).

This maintains the stabilization near the edge of criticality which is consistent with the properties of SOC.

Figure 6 shows the average population dynamics for the worst 5 runs. From an initial standpoint, the overall averages are much higher for the system.

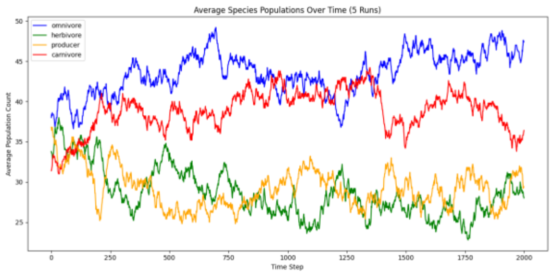

Figure 6: Average Species' Populations Over Time for the Bottom 5 Runs (Determined by the R2 Value)

Here, the herbivores and producers share the same range and seem to be somewhat inversely linked. By having a higher number of producers and herbivores, it is shown that the population of carnivores and omnivores are more stabilised and do not have many significant cascades. This shows that SOC is being suppressed in some way.

Figure 7 shows the energy and biomass averages for these runs. Here it is evident that due to the higher number of producers, the energy average remains consistently higher than in *Figure 5*. This makes it so that the species can reproduce more and allows them to survive longer. With more species, it is much rarer to have cascades due top the fitness being based of the predation traits and number of predators already linked.

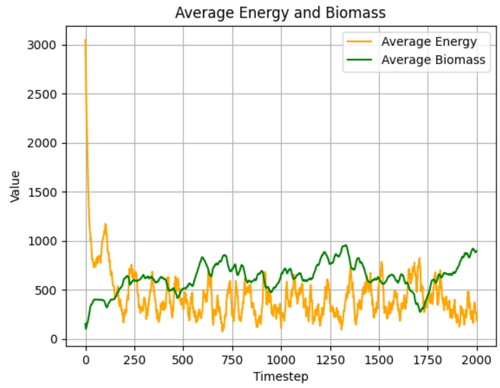

Figure 7: The Average Biomass and Energy for the Bottom 5 Runs (Determined by the R2 Value)

The energetic abundance in the system reduces the likelihood of cascading extinctions, even when one is forced at every time step. This is due to predators having multiple, high-quality prey and therefore reduces the competition aspect of the ecological dynamics, and as an effect, supresses the system's ability to maintain a critical state that is required for true SOC.

4. Discussion

The Bak-Sneppen model *(Bak, P. Sneppen, K. 1993*) gave an abstract insight into co-evolution and its dynamics. This model caused the lowest fitness species to go extinct at each time step, intermittently causing its neighbours to randomly reset. This then caused the system to self-organise into a critical state that was characterised by avalanches and power-law distributions of extinction event sizes.

The dynamic of this model was minimalistic, yet it was effective in producing these scale-invariant events without the requirement of fine-tuning parameters.

The model proposed here extended on the simple foundation provided, attempting to add elements of ecological realism. This included individual agent species, energy-from biomass, and trophic interactions, making use of a directed network with predator-prey links.

These changes made fitness no longer random, instead basing it off traits and connections, similar to real-life examples. It made energy and biomass the constraint of the system dynamics, such as population growth. And it allowed of the network to be dynamic, and change form based upon extinction events and reproductive success.

The extended model had a more adaptive nature than the rigid rules given by the Bak-Sneppen model. In this case, there was no fixed structure or interactions, these were shaped by extinction and reproduction and as a result continually changed agent's fitness. This added a much more dynamic form of natural selection instead of being based off randomness, and as a result was able to maintain a dynamic balance.

The criteria for SOC meant that the system had to follow a power-law distribution, and through a parameter sweep, there were proven to be some iterations that naturally drifted to the edge of chaos, whilst maintaining their stability. These exhibited a perfect power-law fit. These systems also had spontaneous large cascades whilst keeping a stable , yet fluctuating population size – commonly seen with SOC.

The biomass and energy flow showed inverse coupling that suggested a balance between the trophic acquirement and use of energy whilst proving that the producers were responsible for the flow rate.

However, in some simulations, the power-law criteria were not quite met and instead deviated from the scale-invariant behaviour. This perhaps meant that the system stayed below the critical threshold. This could have been due to high average energy that lowered the predative pressure or stabilised the trophic dynamics meaning there were less opportunities for cascades and recovery.

This could have however been slowly moving towards the critical state attractor that was never reached due to the fixed 2,000-time steps given to each simulation, although this would be impossible to confirm.

Therefore, as a safe conclusion it is not guaranteed that SOC can be demonstrated in complex ecological-based models. By adding realism, the system is prone to more constraints which can push it away from the conditions required for SOC to emerge. As mentioned, the system should be resilient to small perturbations in the initial conditions *(Stapleton, M. et al., 200)* and therefore, to be certain a system can exhibit SOC, the power-law requirements should be met every time.

Overall, from the extended model exhibiting signs of SOC in many cases, validates that the Bak-Sneppen foundation to the model was successful. This experiment revealed that ecological accuracy can affect the criticality of these basic systems and shows why criticality is sometimes bounded by its conditions. Some additional rules can help to stabilise criticality while others may need additional mechanisms to prevent divergence from the critical states. *(Zeraati, R., et al., 2020).*

This experiment demonstrates that the Bak-Sneppen model can provide a powerful base for the emergence of SOC. By adding a new set of dynamics, the variability in whether the system demonstrated SOC or not highlights the sensitivity of these systems to complexity. This shows that criticality is never guaranteed in complex systems but instead depends on the conditions chosen.

For future work in this area, this model and its findings could be used to create artificial models of natural ecosystems. By extending the simulation time-steps, and limiting the model to true found data, it may be a way to understand the conditions for self-organised criticality in more depth than abstract models can provide.

References

Bak, P. Sneppen, K. (1993). *Punctuated Equilibrium and Criticality in a Simple Model of Evolution.*

https://doi.org/10.1103/PhysRevLett.71.4083

Bak, P., Tang, C., & Wiesenfeld, K. (1987). *Self-Organized Criticality: An Explanation of 1/f noise.*

https://doi.org/10.1103/PhysRevLett.59.381

Bak, P., Tang, C., & Wiesenfeld, K. (1988). *Self-Organised Criticality.*

https://doi.org/10.1103/PhysRevA.38.364

Clar, S., Drossel, B., Schwabl, F., (1996). *Forest Fires and Other Examples of Self-Organized Criticality*

https://doi.org/10.1088/0953-8984/8/37/004

Dunne, J. A., Williams, R. J., & Martinez, N. D. (2002). *Network Structure and Biodiversity Loss in Food Webs: Robustness Increases with Connectance.*

https://doi.org/10.1046/j.1461-0248.2002.00354.x

Felici, M., Caldarelli, G., Gabrielli, A., Pietronero, L. (2001). *Perturbative Approach to the Bak-Sneppen Model.*

https://doi.org/10.1103/PhysRevLett.86.1896

Grinfeld, M., Knight, P. A., Wade, A. R. (2010). *Bak-Sneppen Type Models and Rank-Driven Processes.*

https://doi.org/10.48550/arXiv.1011.1777

Grimm, V., Railsback, S. F., et al. (2006). *A Standard Protocol for Describing Individual-Based and Agent-Based Models.*

https://doi.org/10.1016/j.ecolmodel.2006.04.023

Hagberg, A. A., Schult, D. A. Swart, P. J., (2008). *Exploring the Network Structure, Dynamics and Function using NetworkX*

https://www.osti.gov/biblio/960616

Holland, J. H. (1992). *Complex Adaptive Systems.*

https://www.jstor.org/stable/20025416

Kutjnak-Urbanc, B., Zapperi, S., Milosevic, S. Stanley, E. H., (1996). *Sandpile Model on Sierpinski Gasket Fractal*

https://doi.org/10.48550/arXiv.cond-mat/9504022

Masad, D., Kazil, J., (2015). *Mesa: An Agent-Based Modelling Framework.*

https://doi.org/10.25080/Majora-7b98e3ed-009

Meester, R. Sarkar, A. (2012). *Rigorous Self-Organised Criticality in the Modified Bak-Sneppen Model.*

https://doi.org/10.1007/s10955-012-0627-2

Plenz, D. Ribeiro, L. T., et al., (2021) *Self-Organized Criticality in the Brain*

https://doi.org/10.48550/arXiv.2102.09124

Stapleton, M., Dingler, M., Christensen, K., (2004). *Sensitivity to Initial Conditions in Self-Organised Critical Systems*

https://doi.org/10.1007/s10955-004-5709-3

Zeraati, R., Priesemann, V. Levina, A. (2020) *Self-Organization Toward Criticality by Synaptic Plasticity*

https://doi.org/10.48550/arXiv.2010.07888